On the Preferred Extensions of Argumentation Frameworks: Bijections with Naive Sets

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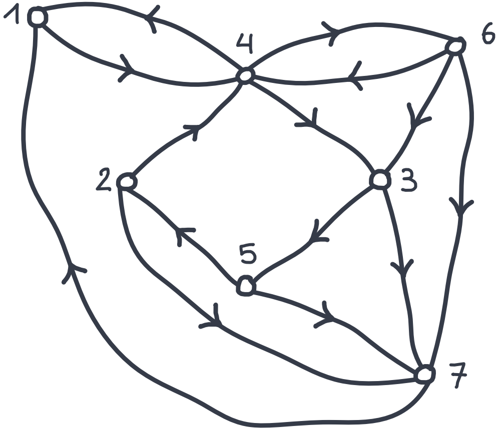
Argumentation frameworks ?



- *Argumentation framework* : arguments *attacking* each others

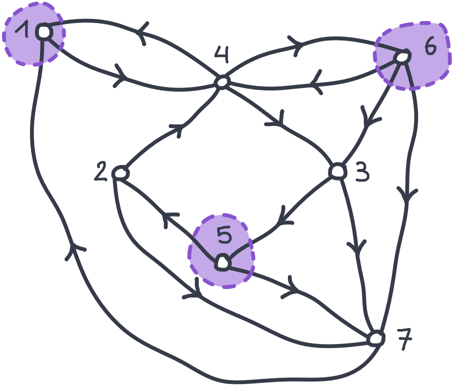
Argumentation frameworks and their use

- Stems from logic, aim to model nonmonotonic reasoning in the presence of unknown/incomplete information (see [Bench-Capon, Dunne, 2007])
- Used in machine learning [Cocarascu, Toni, 2016], decision support systems [Amgoud, Prade, 2009], logic programming [Dung, 1995], ...
- An *argumentation framework* $F = \langle \mathcal{A}, \mathcal{R} \rangle$:
  - a set of *arguments* $\mathcal{A}$ and $\mathcal{R}$ a *binary attack relation* over $\mathcal{A}$
  - can be represented by a *directed graph* (possibly with loops)
- For $S \subseteq \mathcal{A}$:
  - $S^- = \{y \in \mathcal{A} \mid (y, x) \in \mathcal{R}, x \in S\}$, i.e. the arguments *attacking* S
  - $S^+ = \{y \in \mathcal{A} \mid (x, y) \in \mathcal{R}, x \in S\}$, i.e. the arguments *attacked by* S
  - $\Gamma(S) = S^+ \cup S^-$

Admissible sets and extensions

- How to evaluate *"acceptable/persuasive"* groups of arguments in F?
- Some useful concepts, for $S \subseteq \mathcal{A}$
  - *conflict-free*: $S \cap \Gamma(S) = \emptyset$ (independent set)
  - *naive*: S maximal conflict-free (max. ind. set)
  - *self-defending*: $S^- \subseteq S^+$
  - *admissible*: S conflict-free and self-defending
- Respective set systems in F: $CF(F)$, $NAIV(F)$, $SD(F)$, $ADM(F)$
- *Preferred extensions* [Dung, 1995] :
  - *maximal admissible* sets of F, denoted $PREF(F)$
  - not necessarily naive!
  - intuitively : large *"coherent"* and *"relevant"* groups of arguments

Example



- 123 is *naive*, but *not self-defending*
- 2346 is *self-defending*, but *not conflict-free*
- 156 is *preferred*

Enumerating preferred extensions

Problem. Enumerating Preferred Extensions (EPR)

In: an argumentation framework $F = \langle \mathcal{A}, \mathcal{R} \rangle$

Out: the set $PREF(F)$ of preferred extensions of F

- Deciding the existence of a non-trivial preferred extension is NP-*complete* [Dimopoulos, Torres, 1996]
- So, EPR cannot be solved in output-polynomial time *unless* P = NP
- *Polynomial-delay* for some classes of argumentation frameworks [Coste-Marquis et al., 2005]

Strategy: use naive sets

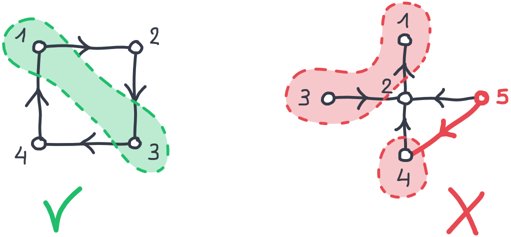
- Enumerating naive sets: *polynomial delay* [Johnson et al., 1988]
- **Idea.** use naive sets to enumerate preferred extensions [Dunne et al., 2015]!
- Increasingly demanding approaches:
  1. cases where $PREF(F) = NAIV(F)$
  2. find another framework F' such that $PREF(F) = NAIV(F')$
  3. find another framework F' inducing a *bijection* between $PREF(F)$ and $NAIV(F')$

First approach. Identifying cases where

$$PREF(F) = NAIV(F)$$

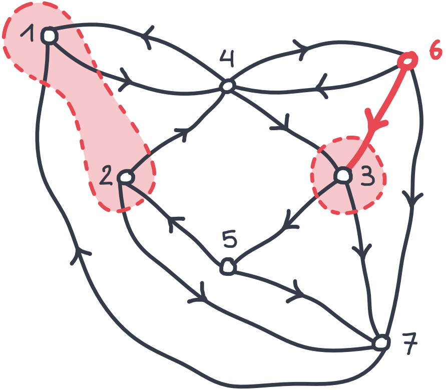
Naive-bijective argumentation frameworks

Definition. An argumentation framework F is *naive-bijective* if $PREF(F) = NAIV(F)$.



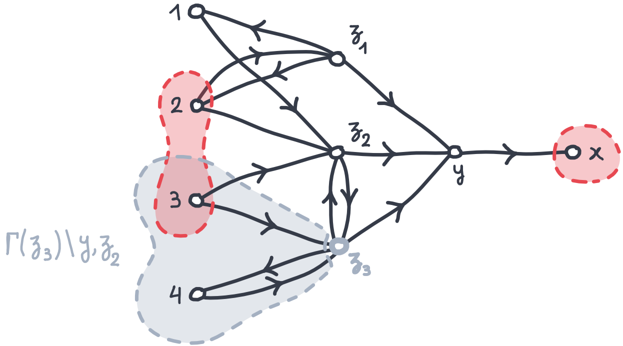
- Applies to *symmetric argumentation frameworks* [Coste-Marquis et al., 2005]
- **Question.** How to recognize naive-bijective argumentation frameworks?
- **Remark.** if $x \in \mathcal{A}$ cannot be defended and $(x, x) \notin \mathcal{R}$, answer is *no*

Running Example



- 123 is naive, but *does not defend itself* against 6
- **Remark.** F fails naive-bijectivity $\Leftrightarrow$ some naive-set set is not self-defending

Why is naive-bijectivity failing?



- $34x$ conflict-free but needs to defend x against y
  - $34x \cap \Gamma(z_1) = \emptyset \implies 34xz_1$ is a *conflict-free solution*!
- $23x$ conflict-free but needs to defend x against y
  - $23x$ *transversal* of $\{\Gamma(z_1), \Gamma(z_2), \Gamma(z_3)\} \implies$ *no conflict-free solution*

Recognition and properties

- Some pre-processing: if x cannot be defended, add (x, x) to $\mathcal{R}$
- F is not naive-bijective $\Leftrightarrow$ some $S \subseteq \mathcal{A}$ does not defend x (x in S) against y and:
  - S is *conflict-free* in F
  - S is a *transversal* of $\{\Gamma(z_1), \Gamma(z_2), \ldots, \Gamma(z_m)\}$, $z_i \in y^-$

Theorem. Let F be an argumentation framework. Deciding that F is *naive-bijective* is *co*NP*-complete*.

Proposition. Let F be an argumentation framework. If F has *constant in-degree*, then checking *naive-bijectivity* can be done in *polynomial time*.

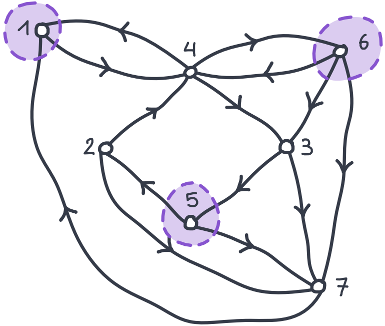
Second approach. Find another framework F' such that $PREF(F) = NAIV(F')$

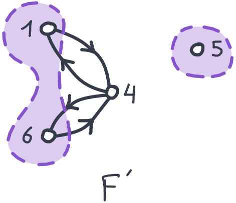
Naive-recasting

Definition. [Dunne et al., 2015] An argumentation framework F is *naive-recasting* if there exists another framework $F' = \langle \mathcal{A}', \mathcal{R}' \rangle$ such that $PREF(F) = NAIV(F')$.

- Recognizing naive-recasting frameworks is NP-*complete* [Dunne et al., 2015]
- For instance, naive-bijective frameworks
- **Question.** Can we find a naive-recasting class of argumentation frameworks?

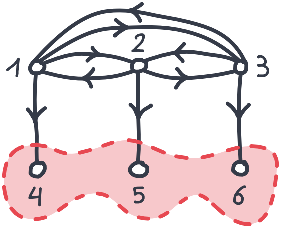
Running example



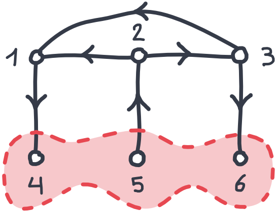


- $PREF(F) = \{45, 156\}$
- **Question.** Are 45, 156 the naive sets of some F'? *Yes!*
- **Remark.** conflicts of F *reflects* in F'

Negative example



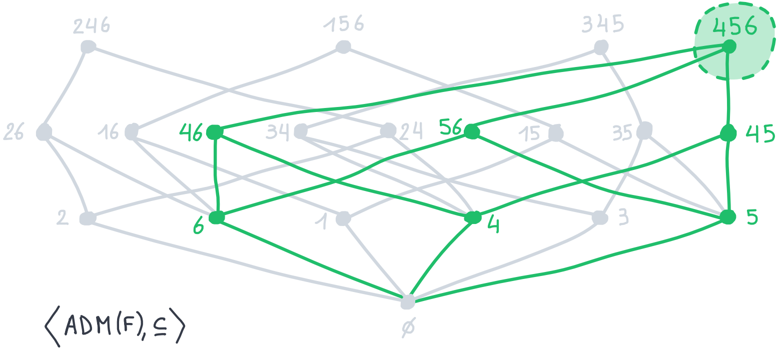
F



F'

- $PREF(F) = \{345, 246, 156\}$. Is F naive-recasting?
- 456 *conflict-free* in F implies 456 *conflict-free* in F'
- But then, $456 \in NAIV(F') \setminus PREF(F)$ ... so F *cannot be naive-recasting*

Admissible sets and naive-recasting



- Admissible sets *do not capture* 456, they *lack structure*...
- **Idea.** close under *intersection* and use *conflict-free/self-defending sets*:
  - 345 conflict-free, 4 and 5 self-defending $\Longrightarrow$ 45 *admissible*
- 45, 56, 46 conflict-free implies 456 conflict-free and hence *admissible*!

Admissible-closed argumentation frameworks

Definition. An argumentation framework F is *admissible-closed* if for every two admissible sets A_1, A_2, their intersection $A_1 \cap A_2$ is also admissible.

- the union of two self-defending sets is self-defending
- Assuming F is admissible-closed :
  - for each $S \in PREF(F)$, $\{A \in ADM(F) \mid A \subseteq S\}$ is closed under union and intersection $\implies$ corresponds to a *distributive lattice*
  - for every $A_1, A_2, A_3 \in ADM(F)$,

$$A_1 \cup A_2, A_1 \cup A_3, A_2 \cup A_3 \in ADM(F) \text{ implies } A_1 \cup A_2 \cup A_3 \in ADM(F)$$

- $\langle ADM(F), \subseteq \rangle$ is a *median semilattice* [Barthélemy, Constantin, 1993]

Consequences and Results

> **Theorem.** Every admissible-closed argumentation framework is naive-recasting.

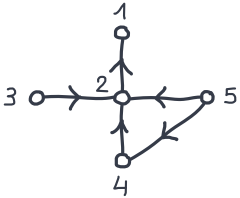
- However, using similar arguments as for naive-bijectivity ...

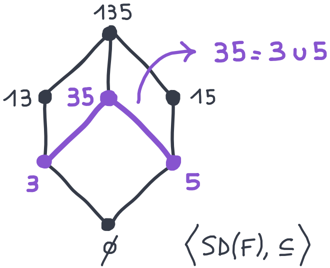
> **Theorem.** Deciding that an argumentation framework is admissible-closed is NP-*complete*.

Third approach. Find a framework F' inducing a bijection between $PREF(F)$ and $NAIV(F')$

Finding a bijection

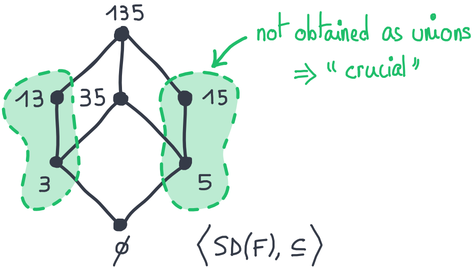
- **Idea.** use the *lattice structure* of self-defending sets [Dung, 1995]
- Let $SD(F)$ be the self-defending sets of F:
  - $\emptyset \in SD(F)$ and $S_1, S_2 \in SD(F)$ implies $S_1 \cup S_2 \in SD(F)$
  - so, $\langle SD(F), \subseteq \rangle$ is a *lattice*.





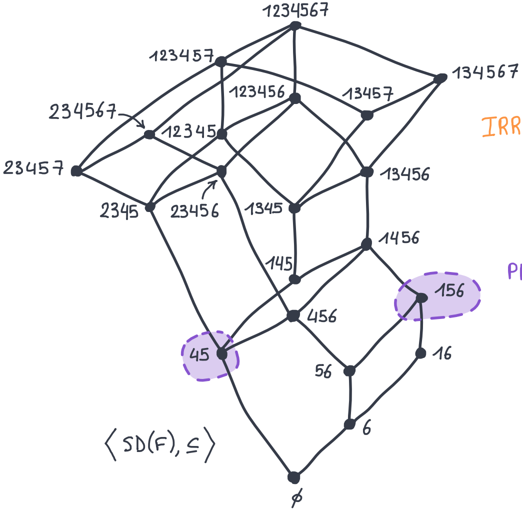
Irreducible self-defending sets

- $SD(F)$ is generated by taking unions of self-defending sets ...
- but which ones are important ?



Definition. Let F be an argumentation framework. A set $S \in SD(F)$ is *irreducible* if $S \neq \emptyset$ and for every $S_1, S_2 \in SD(F)$, $S = S_1 \cup S_2$ implies $S = S_1$ or $S = S_2$. We denote $IRR(F)$ the set of irreducible elements of $SD(F)$.

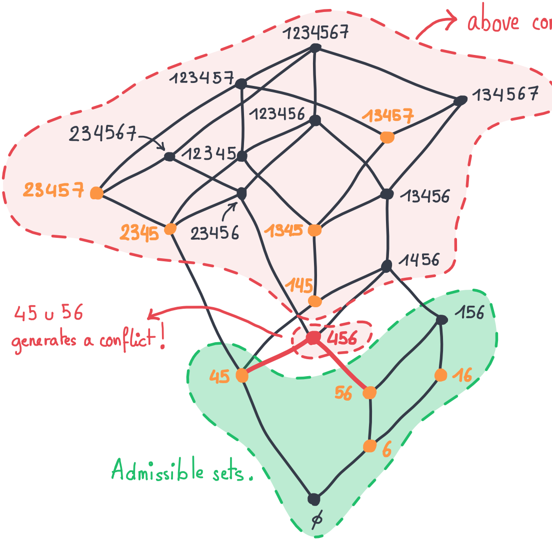
Running example

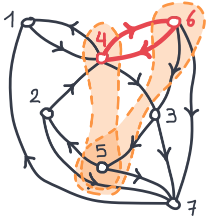


IRR(F) = {6, 45, 56, 16, 145, 1345, 2345, 23457, 13457}

PREF(F) = {45, 156}

Reasoning with irreducible self-defending sets



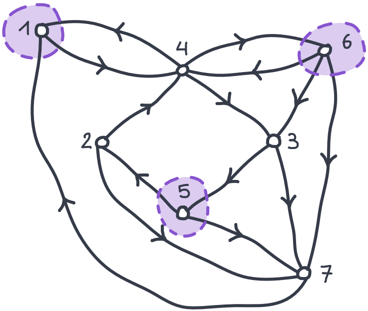


Reasoning with irreducible self-defending sets

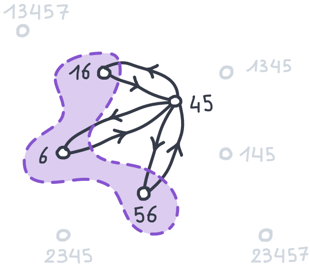
- a self-defending set S is *characterized* by irreducible elements:
  - $IRR(S) = \{S' \in IRR(F) \mid S' \subseteq S\}$
  - $S = \bigcup IRR(S)$
- Hence, S is not conflict-free *if and only if*:
  - some $S' \in IRR(S)$ is *not conflict-free* alone, or
  - some pair $S_1 \cup S_2$ is *not conflict-free*, with $S_1, S_2 \in IRR(S)$
- Consider the framework $F_{IRR} = \langle \mathcal{A}_{IRR}, \mathcal{R}_{IRR} \rangle$ where:
  - $\mathcal{A}_{IRR} = \{S \in IRR(F) \mid S \text{ is conflict-free}\}$
  - $\mathcal{R}_{IRR} = \{(S_1, S_2) \mid S_1 \cup S_2 \text{ is not conflict-free}\}$

Theorem. Let F be an argumentation framework. A set $S \in SD(F)$ belongs to $PREF(F)$ *if and only if* $S = \bigcup N$ for some $N \in NAIV(F_{IRR})$. This correspondence is *bijective*.

Running Example



$156 = 16 \cup 6 \cup 56$



PREF(F) = {45, **156**}

$F_{IRR} = \langle \mathcal{A}_{IRR}, \mathcal{R}_{IRR} \rangle$

Results

Theorem. Let F be an argumentation framework. If $IRR(F)$ is given, $PREF(F)$ can be enumerated with *polynomial delay*.

- **Question.** If $IRR(F)$ is not given, can it be enumerated from F?
  - *harder than hypergraph dualization* in general [Khardon, 1995]
  - but *polynomial* (in the size of F) for some classes of lattices [Wild, 2017]!

Conclusion

- **Problem.** given F, enumerate the sets of $PREF(F) \implies$ *hard* problem
- Use *naive sets*:
  1. cases where $PREF(F) = NAIV(F)$
  2. *recasting*: find some F' such that $PREF(F) = NAIV(F')$
  3. find some F' with a *bijection* between $PREF(F)$ and $NAIV(F')$
- Results:
  1. *hard to recognize*, except for frameworks with *constant in-degree*
  2. *admissible-closed frameworks* are recasting, but *hard to recognize* too...
  3. *polynomial-delay* algorithm based on a *bijection* with *irreducible self-defending sets*
- Further works:
  - computing F' if F is admissible-closed?
  - frameworks where $IRR(F)$ can be computed easily?

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